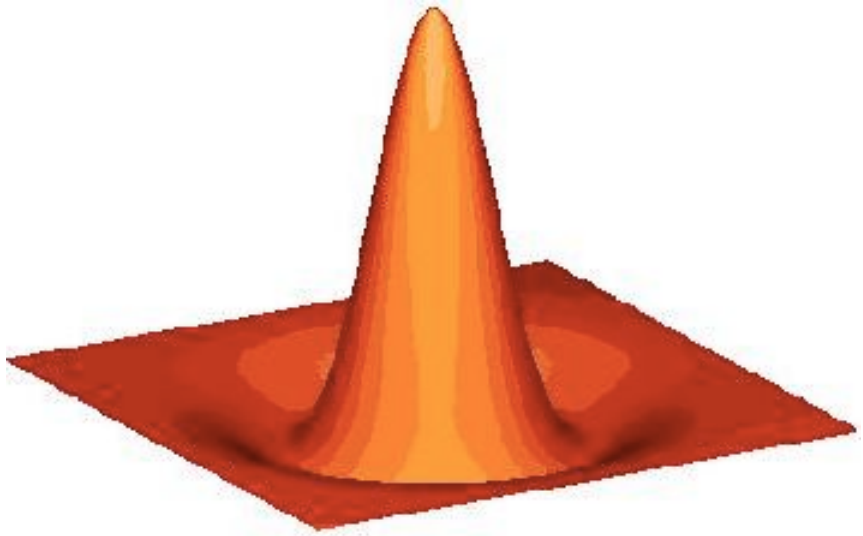
Source Detection I:Theory



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3rd X-ray Astronomy School

14 May 2003

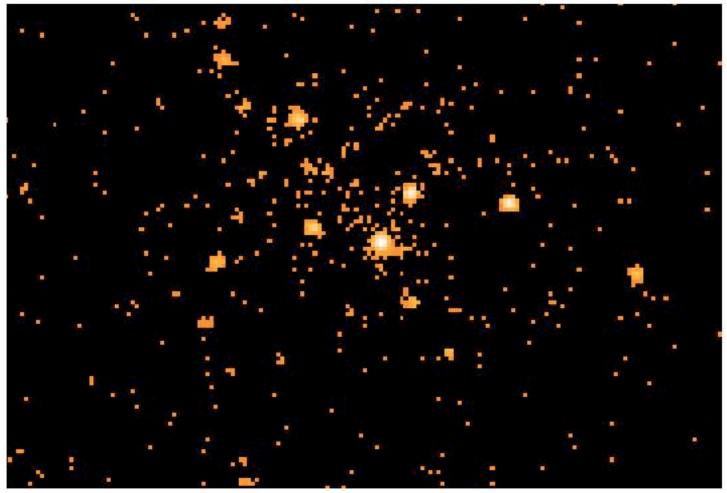
The Challenge: Source Detection

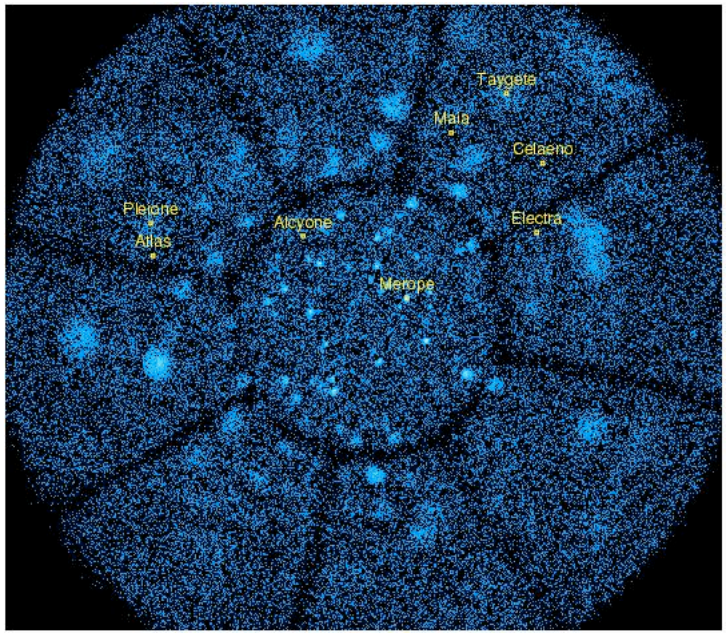
What challenge??

(Granted, it does look easy here. This is a *Hubble* image of the globular cluster 47 Tuc, courtesy P. Edmonds.)

The Challenge: Source Detection

It gets harder: not everything seen in this false-color *Chandra* image of the core for 47 Tuc is an X-ray source. (P. Edmonds)

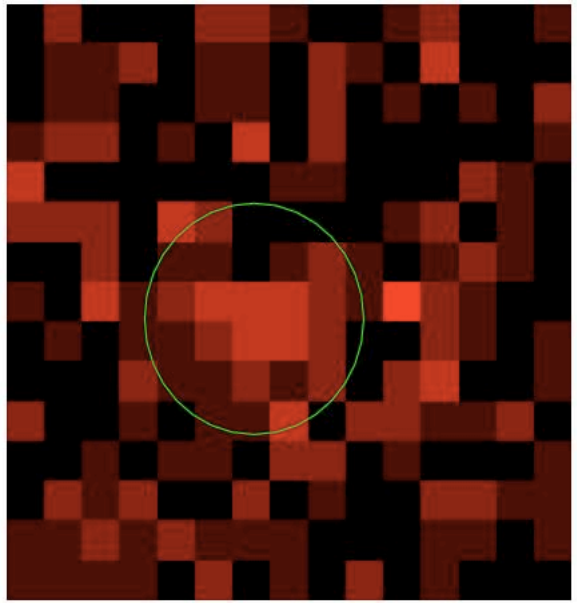


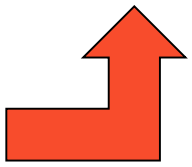


As seen in this *ROSAT* image of the Pleiades, the X-ray source detector must worry about high backgrounds and exposure variations (the edge of the circular field-of-view and the support-rib shadows). (V. Kashyap)

The Challenge: Source Detection

- Sources may be easy to see in a typical *Hubble* image. However, detecting and characterizing them becomes increasingly difficult at higher energies.
- Source data may consist of only a few counts, hence we must rely on the Poisson distribution when making statistical inferences.
- Some spatially extended sources (*e.g.*, supernova remnants) emit brightly at high energies and may overlap with point sources, making detection and characterization of the latter more difficult.
- How a high-energy telescope blurs a point source (*i.e.*, the telescope's point-spread function) may be spatially non-uniform.





- *Is this a source, or a background fluctuation*?

Detection Theory: the Short Form

Note: the following statements are generic; they may or may not apply to the specific source detection algorithm of your choice. They are meant to build your intuition.

- *The Ingredient(s):* a *N*-dimensional event list or binned "image." (And an exposure map, knowledge of the PSF, *etc*., if appropriate.)
- *The Detection "Tool":* some function that is localized (*i.e*., non-zero only over some characteristic scale) within at least some subset of the dimensions.
- *The Hypotheses:*

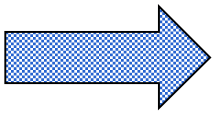
  M_0: the data in a given pixel are (Poisson-)sampled from the background.

  M_1: the data are a sum of samples from the background and an astronomical source.

The Five-Fold Path

For a given source detection algorithm, an analyst might follow this five-fold path to source detection Nirvana:

- Select an appropriate function scale, _. (If one is attempting to detect a point source, this would be some encircled-energy radius of the PSF.)
- Estimate the background amplitude, B. (In actuality, one would do this estimation for each image pixel-here, we narrow the problem to a single pixel.)
- Determine the value of a selected model comparison test statistic, T_o.
- Determine the significance, _: $\alpha = \int_{T_o}^{\infty} dT\, p(T|f[\hat{B}])$
- Compare _ to a pre-determined threshold significance value $\__o$.



If _ < $\__o$, the pixel is associated with a source!

Classic Detection: CELLDETECT

- *The Function(s):* two box functions with unit amplitude, co-aligned and centroided at pixel (*i,j*). The number of counts within each box are D_d and D_b.
- *The Determination of B:* done by assuming (a) the truth of the alternative model, and (b) that the source is point-like:

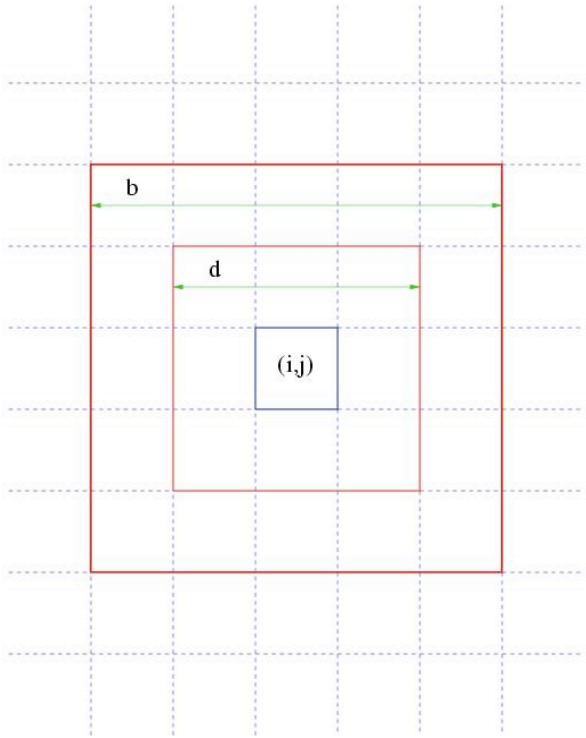
$$D_d = \alpha\hat{S} + \hat{B}$$
$$D_b = \beta\hat{S} + \left(\frac{b}{d}\right)^2 \hat{B}$$

where _ and _ are the integrals of the PSF within each box, respectively.

- *The Model Comparison Test Statistic:* the signal-to-noise ratio, or *SNR*:

$$T_o = \hat{S}/\sigma_{\hat{S}}$$

- *Associating a Pixel with a Source*: If $SNR > SNR_{thr}$, accept the alternative model.
- For more information: *CIAO* detect manual.



New Detection: WAVDETECT

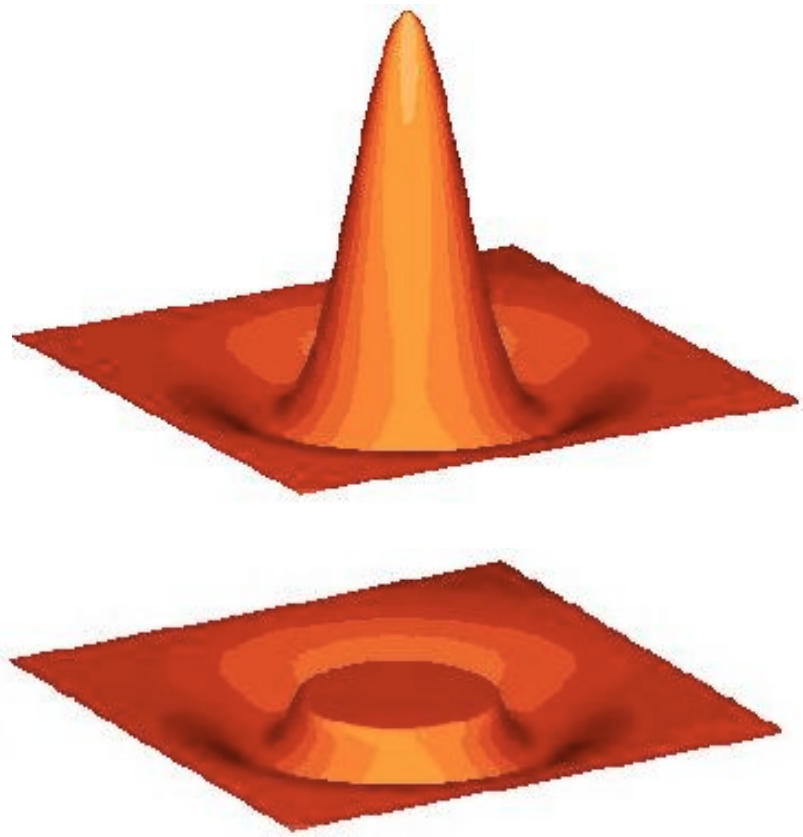
- *The Function*: the Marr, or "Mexican Hat" wavelet, $W(\_)$ (*above, right*), which is non-zero within a circle of radius $\sqrt{5}\_$ from the centroid.
- *The Determination of B*: done by determining the average number of counts per pixel in the wavelet negative annulus (*below, right*), while using it as a weighting function; done iteratively, with source counts removed from the field until the background estimate stabilizes.
- *The Model Comparison Test Statistic*:

$$T_o = C_o = \_{i'} \_{j'} W_{i-i',j-j'} D_{i'j'}$$

- *Associating a Pixel with a Source*: if

$$\_ = \int_{C_o}^{4} dC\, p(C|2\pi\_^2 B) < \_o$$

accept. A typical choice for the threshold is $1/P$, where P is the number of pixels examined in the image; it thus corresponds to a number of false pixels.

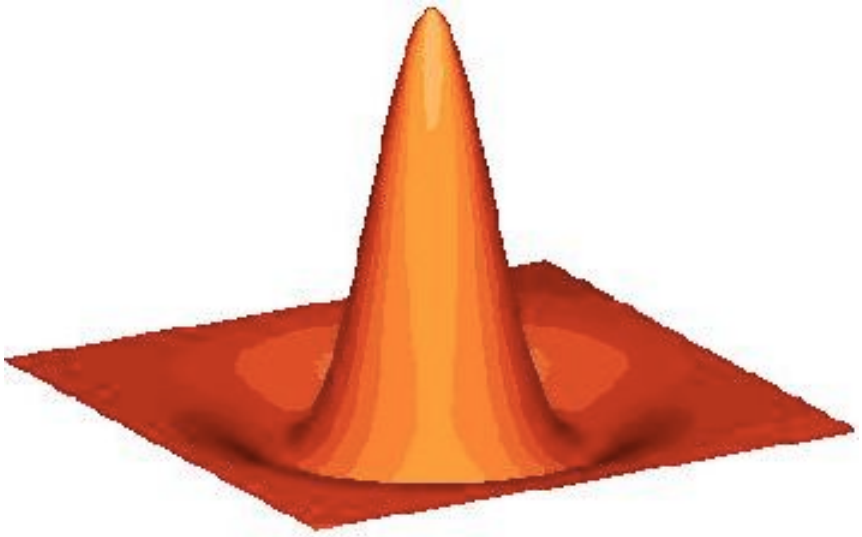


See Freeman et al. 2002, ApJS 138, 185 for more details.

Why Mexican-Hat Wavelets?

- The Gaussian-like positive kernel has a shape similar to canonical point-spread functions (PSFs).
- The function is localized in both the spatial *and* Fourier domains; a dyadic (factors of two) sequence of scales is sufficient to sample the frequency domain.
- It has two "vanishing moments": the correlation of MH with constant and linear functions is zero. It thus "annihilates" the contribution of a spatially constant or linear background to the correlation coefficients.

$$W\left(\frac{x}{\sigma}, \frac{y}{\sigma}\right) = \left[2 - \frac{x^2 + y^2}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

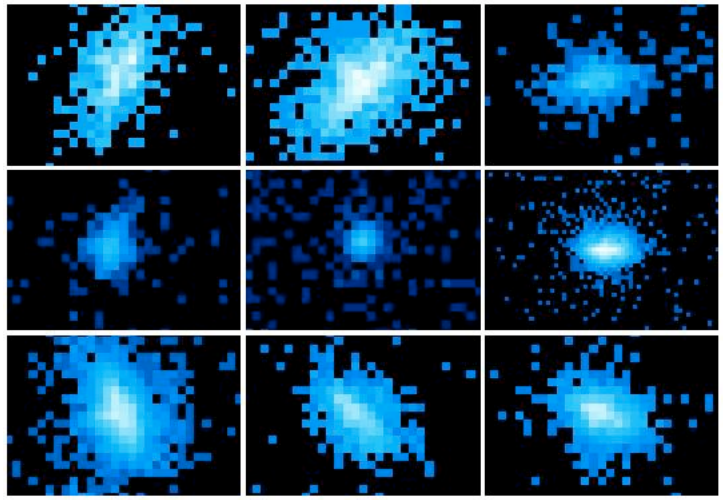


Potholes on the Five-Fold Path

- PSFs in the X-ray regime are spatially varying (which partially motivates the multi-scale approach to source detection; the other major motivation is the study of extended sources, *e.g.*, SNRs, hot gas in clusters).
- The optimal determination of B from raw data consisting of source and background counts is an unsurmounted statistical challenge.
- The cosmic background is not necessarily spatially constant!
- The probability sampling distributions (PSDs) from which observed values of T are sampled generally cannot be represented analytically, except asymptotically in the high-counts limit; simulations are needed.
- There is no model comparison test statistic T that has been proven to be "most powerful"…and test power is extremely difficult to compute.
- And exposure maps, vignetting, *etc*.: Vinay speaks of these.
- Below, I expand on some of these issues…

Pothole: Spatially Varying PSFs

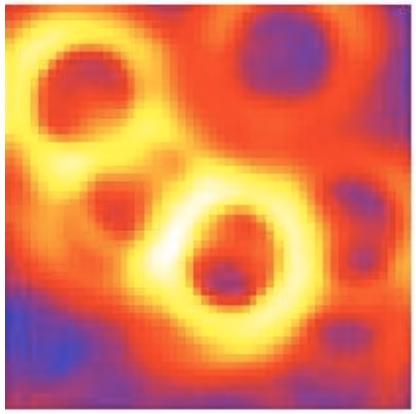
A point source observed on-axis (center) with an X-ray telescope will be more sharply in focus than a source observed off-axis (outer eight panels), in large part because the counts-recording instruments are flat. Sources detected using a cell or wavelet of one scale may not be detected at another scale: a multi-scale approach is necessary for robust detection!

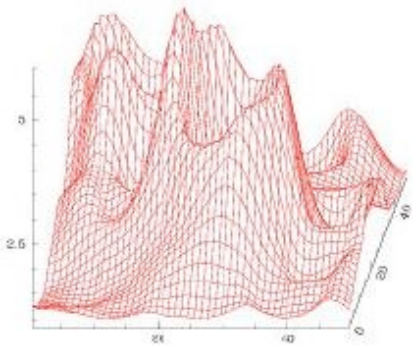


Pothole: Background Determination

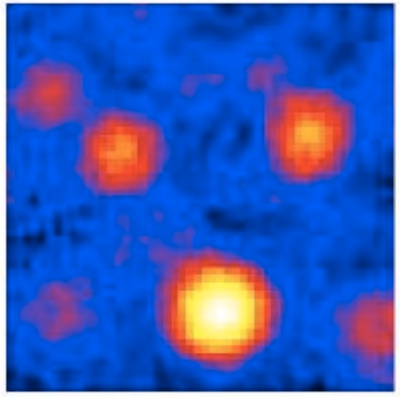
- In theory, can model cosmic+particle background, but not easily done.
- Estimated from raw data. How to do?
- If one uses PSF information, one must make assumptions of spectral form (since width varies as function of energy); also, bad for detecting extended sources.
- WAVDETECT computes backgrounds at each scale, and combines them; accurate enough for source detection, but systematic rings (*top right*) and bumps (*bottom right*) make final result not necessarily quanitatively accurate.

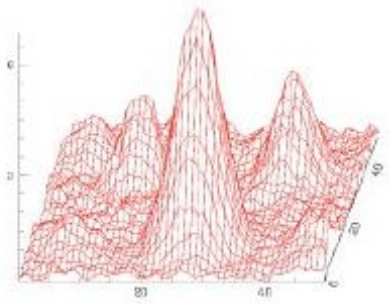
Strong source biases background in ring around it:





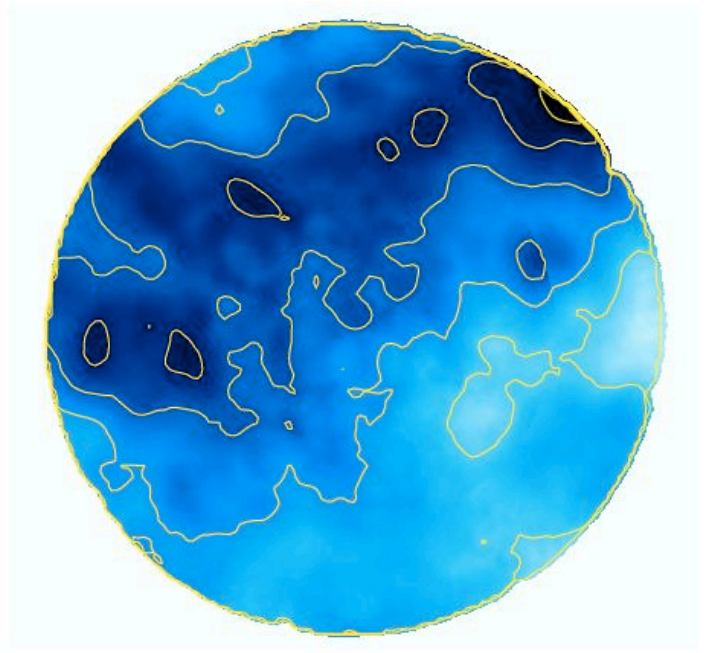
Large-scale source biases background at source location:





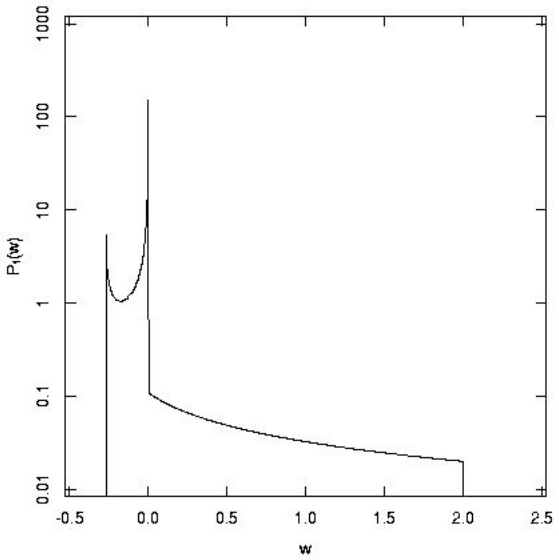
Pothole: Spatially Varying Backgrounds

Nearby regions of dense gas/dust can absorb the cosmic background (e.g., from AGNs), creating X-ray "shadows" such as that observed in the Pleiades. (Nearby emission in the hot local bubble means that we still see background photons in shadowed regions.)



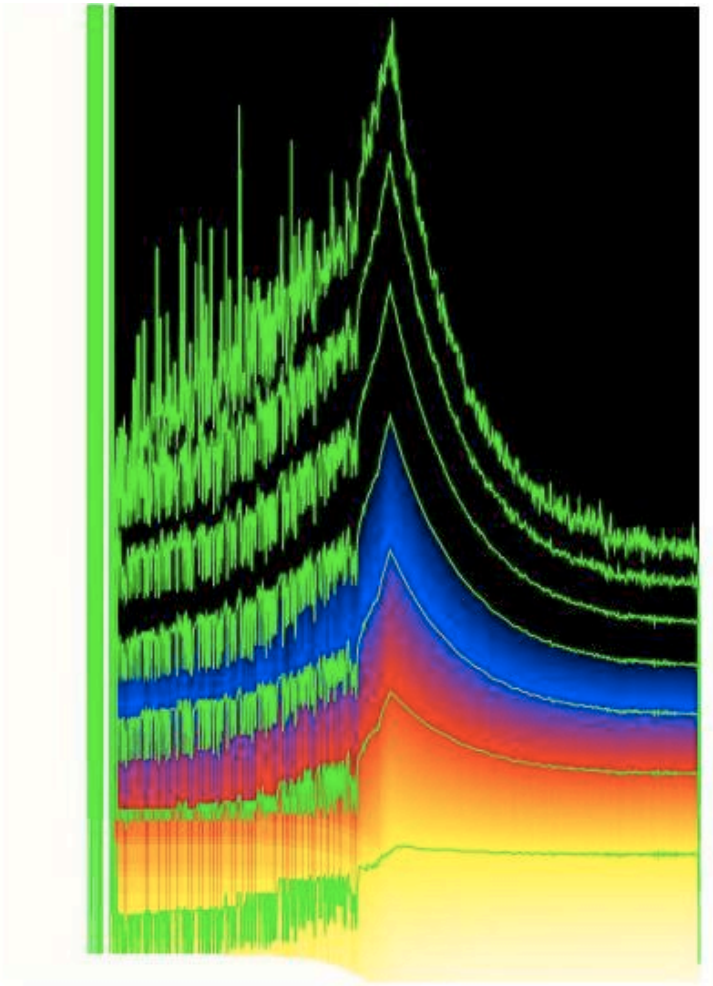
Pothole: Computation of Significance

- In the high-count limit, $p(C|q = 2\pi B^2)$ tends to a zero-mean Gaussian distribution of width $\_ = q^{1/2}$.
- Elsewhere, $p(C|q)$ is determined via simulations. (At right, a sample PSD at low counts from Damiani *et al.* 1997.)



Pothole: Computation of Significance

The picture to the right shows the significance (from simulations) as a function of *q* and *C* (to the left of the cusp) or *C/sqrt(q)* (to the right of the cusp). The contours are 0.1 (bottom) to 10^{-7} (top). This figure demonstrates that a relatively smooth distribution (asymptotically Gaussian, at high *q*) becomes very messy at low *q*, and shows that simulations are required. The low *q* limit is important for *Chandra*, whose the smaller field of view greatly reduces the number of cosmic background events per pixel per second, relative to *ROSAT*.



Pothole: Type II Error

- The Type II error is nearly impossible to compute for current source detection algorithms because of the fuzzy way the problem is stated: the alternate hypothesis is that "pixel *(i,j)* includes some number of source counts."
- Computed instead is the "detection efficiency": how often does the algorithm detection a source of strength *x*, at off-axis location *y*, when the background is *z*…
    - Unlike Type I error, detection efficiency is instrument-specific.
    - Depends on scale sizes, background, amplitude, extent, spectrum, and off-axis angle, in addition to the details of the exposure at the source location.
- A related topic: upper limits ("I don't detect a source at *(i,j)*. How strong could an underlying source be and still not be detected?"). Rarely analytically computable, it can be read off from detection efficiencies, if those have been computed.